

- (ii) Lattice Approximations to the Free Euclidean Field corrupted by noise.
- (iii) Lattice Approximation to the Free Euclidean Field coupled to a Yang-Mills Field observed in noise.

Computing the conditional distribution of the field conditioned by noise gives rise to Gibbs random fields with a random external field.

Understanding the order-disorder phenomena for these systems is a topic of current interest in mathematical physics. The concept of temperature has a natural meaning in the context of estimation of random fields, and the problem of estimation of parameters such as temperature from noisy observations is also discussed.

In the context of these problems we discuss the concept of Stochastic Quantization (as originally proposed by Parisi and Wu) and show its analogy to the relationship between Wiener and Kalman Filtering Problems.

The actual computation of these estimates can be done using Monte Carlo Methods such as the Metropolis Algorithm or Simulated Annealing. Some results on the asymptotic analysis of simulated annealing are presented. Moreover, the solutions of these estimation problems can be implemented in a distributed architecture.

In the final part of this lecture I show how ideas of Scattering Theory as developed by Adamjan and Arov combined with ideas from Stochastic Systems Theory can lead to solutions of problems in Statistical Physics.

Minimizing the Expected Time to Reach a Goal

Steven Orey, *University of Minnesota, USA*

The results below were obtained in joint work with D. Heath, V. Pestien, and W. Sudderth.

Consider the following control problem: On the interval $(0, 1]$ the process $X(t)$ is given by

$$X(t) = x + \int_0^t X(s)[\mu(s) ds + \sigma(s) dW(s)]$$

where W is a standard Wiener process and μ and σ are non-anticipating controls to be chosen subject to $(\mu(t), \sigma(t)) \in \mathcal{S}$, where \mathcal{S} is a specified subset of $\mathbb{R} \times \mathbb{R}_+$, and the object is to minimize the expectation of $T = \inf\{t: X(t) = 1\}$. Denoting the desired infimum by $V(x)$ (the value function) it was shown by Pestien and Sudderth [Continuous-time Red and Black: How to control a diffusion to a goal, to appear in *Math. of O.R.*] that if $M = \sup\{\mu - \sigma^2/2: (\mu, \sigma) \in \mathcal{S}\}$ then $V(x) = -(\log x)/M$ and if $M = \mu_0 - \sigma_0^2/2$ for some $(\mu_0, \sigma_0) \in \mathcal{S}$ then $\mu(t) \equiv \mu_0$, $\sigma(t) \equiv \mu_0$, $\sigma(t) \equiv \sigma_0$ is optimal, provided $\lambda\mathcal{S} \subseteq \mathcal{S}$ for $0 \leq \lambda < \infty$. Later Heath and Sudderth showed the result remains correct provided $\lambda\mathcal{S} \subseteq \mathcal{S}$ for $0 \leq \lambda \leq 1$. Here we show the result is correct if and only if $M < \infty$ and $I = \inf_{\varepsilon > 0} \sup\{\mu - (\frac{1}{2} - \varepsilon)\sigma^2: (\mu, \sigma) \in \mathcal{S}\} < \infty$.

In their original work (cited above) Pestien and Sudderth introduced a very general ‘verification theorem’ for control problems. This still is insufficient to handle

the problem discussed above, but further extensions are given which turn out to be adequate for the problem at hand, and which should be of independent interest.

Related problems and applications are also discussed.

A Brief Survey of Quantum Stochastic Calculus

K.R. Parthasarathy, *Indian Statistical Institute, New Delhi, India*

The present work is in collaboration with R.L. Hudson. It is observed that the classical Brownian motion and Poisson process can be expressed entirely in terms of Weyl representation of the inhomogeneous Euclidean group of the Hilbert space $L_2[0, \infty)$. Such a view permits the extension of the notion of stochastic integrals with respect to 'quantum Brownian motion' and 'quantum Poisson point process'. This leads to a quantum Ito's formula. In particular, the classical Ito's formulae for Brownian motion and Poisson process turn out to be consequences of the canonical commutation rules of the Bose-Einstein field.

The notion of a quantum stochastic differential equation is introduced and a 'Schrödinger equation in the presence of noise' is solved. Furthermore, the fermion field operators are explicitly realized in $L_2(P)$, P being Wiener measure.

On Stochastic Partial Differential Equations

Yu. A. Rozanov, *Steklov Mathematical Institute of Academy of Sciences, Moscow, USSR*

Quantum Theory and Stochastic Processes — Some Contact Points

L. Streit, *Universität Bielefeld, FR Germany*

In recent years progress in both fields has frequently been inspired by a cross-disciplinary exchange of problems, ideas, and results. We first give an overview of these developments and then take a closer look at two examples: 'Dirichlet Forms and Schroedinger Theory' and 'Hida Calculus and the Feynman Integral'.

A 'Propagation of Chaos' Result for Burger's Equation

A.S. Sznitman, *Université de Paris VI, France*

We first discuss the construction of a system of N particles on R satisfying

$$dX_t^i = dB_t^i + \frac{c}{N} \sum_{j \neq i} dL^0(X^j - X^i)_t, \quad i = 1, \dots, N,$$